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NUMERICAL MODELING OF THE QUASISTATIONARY REGIME OF ERUPTION OF A VOLCANO AT THE INITIAL STAGE

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Under the assumption of axial symmetry, the quasistationary phase of the initial stage of a volcanic eruption is modeled. The constructed model allows calculation of the distribution of the regimes of magma flow in the volcanic channel and the quasistationary distribution of a suspension of gas-dust volcanic ejections in the atmosphere from the prescribed parameters of the magmatic source. The model can be used for evaluation of the parameters of the atmosphere above an erupting volcano, the height of a dust column during the quasistationary phase of eruption, the thickness of the layer of ash fallen on the earth, and other forms of action of volcanic eruptions on the environment.

Introduction. A volcanic eruption is one of the most destructive types of natural disasters. We know about the horrific effects of the eruption of such volcanos as Vesuvius (A.D. 79), which destroyed the towns of Pompeii and Herculaneum, Thira (formerly Santorin) (1650 B.C.), which destroyed Minoan civilization, Krakatau, which ejected about 1000 cubic kilometers of magma (1883, energy of eruption about 370 Mtons in TNT equivalent), or Mount Tambora, which took 92,000 lives (1815, energy of eruption 25 Gtons). Although eruptions of this power usually happen once every several decades or centuries, their environmental effects and possible scale of destruction are so great that evaluations of their consequences and the pattern of the process are an extremely important and topical problem.

The influence of volcanic eruptions on the environment is very diverse. It includes direct destruction of the adjacent regions (to 1000 km^2 for large-scale eruptions) and the formation of gas-dust clouds enveloping the earth and not only being a severe hazard to aviation but also changing the gas composition of the atmosphere substantially, which can result in local and global (for example, in prehistoric times) climatic changes [1]. Eruptions of gas-saturated magmas expressed as the explosive or continuous flow of gas-dust jets out of the volcanic neck present the greatest hazard to man. Unlike purely lava eruptions, the velocity of propagation of their products is much higher, while the flow rate of magma can experience changes of orders of magnitudes with transition to a catastrophic stage during a very short period.

It is well known [2–5] that volcanic eruptions usually begin with small ejections of gas and lava but, at subsequent instants, the process can become explosive; at the explosive stage, dust, volcanic ash, and gas consisting of steam and different compounds of sulfur, nitrogen, carbon, and chlorine are ejected into the atmosphere in large amounts. The type of volcanic eruption is largely determined by the presence of dissolved volatile components in the magmatic melt which separate into the free phase, as the magma rises to the surface, and by the viscosity of the magmatic melt. Depending on these parameters, eruption can vary from a purely lava type to a purely gas type with a continuous or explosive ejection of a gas-dust mixture.

Quantitative evaluations of the influence of volcanic eruptions on climatic changes on the earth are given, for example, in [6–10], and they demonstrate that large-scale eruptions can result in global changes on the planet.

Attempts at modeling mathematically the processes involved in volcanic activity on the earth have been made recently in a number of works. In this connection, noteworthy is [11], in which the method of prediction of the propagation of volcanic aerosol ejections into the atmosphere has been set out, and the series of investigations of Barmin and Mel'nik [12–16] into the modeling of volcanic eruptions of high-viscosity, gas-saturated magmas. An attempt at modeling numerically the gasdynamic processes in the atmosphere after a high-power volcanic explosion was made for

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the first time in [17]. The authors assumed that the gas and dust are concentrated in the hemisphere at the initial instant of time and then the gas mass heated to 1700 K is expanded at an excess pressure of the gas of 420 atm. This corresponds to the volcanic explosion of Krakatau with an energy of 370 Mtons [18].

A model of a volcanic explosion in which consideration is given to the propagation of the shock wave formed and the motion of a dust cloud has been proposed in [19]. The results of mathematical modeling of jet eruptions with prescribed values of the physical parameters in the outlet cross section of the volcanic neck have been presented in [20].

In [21], we have modeled mathematically the behavior of a gas and a dust in the atmosphere in the upper part of the volcano's neck in the initial stage of explosive eruption. From the results of calculations we have found the empirical dependence of the amplitude of the shock wave on the distance to the epicenter of the eruption. The general pattern of change in the concentration of the dust near the volcanic crater is given, and the dependence of the height of the gas-dust column on the explosion energy is investigated, as is the dynamics of ejection of ash in volcanic eruption.

In the present work, we model the quasistationary phase of the initial stage of a volcanic eruption, i.e., the period of time where the parameters of a magmatic source change slowly as compared to the processes in the volcanic channel and in the atmosphere. Not only is the flow of eruption products in the atmosphere considered but also the stationary distribution of different regimes of magma flow in the volcano's neck. The problem is solved in a cylindrically symmetric formulation.

Physicomathematical Model of Motion of Magma. The volcanic system will be modeled by a source with a prescribed pressure in it and a cylindrical channel of constant cross section with rigid walls (the channel connects the source with the earth's surface). We will consider magma to be a Newtonian fluid with a viscosity dependent on the concentration of the dissolved gas and the temperature [16]:

$$\mu = \mu_0 \exp\left[-\frac{A}{RT}\exp(-Bc_{\mathrm{m}}) - 1\right], \quad \mu_0 = 10^4 - 10^9 \quad \mathrm{Pa{\cdot}sec}\,. \tag{1}$$

The relationship between the pressure of the gas dissolved in the magma and the equilibrium value of the mass concentration is given by the formula [22]

$$c_{\mathrm{m}} = k_{\mathrm{s}}\sqrt{p_{\mathrm{m}}/p_{\mathrm{a}}}\,, \quad k_{\mathrm{s}} = (13 - 20)\cdot 10^{-3}\,. \tag{2}$$

According to [16], flow in the volcanic channel can generally be subdivided into five regions. In the lower region (region 5 in Fig. 1), where the dissolved-gas pressure exceeds the saturation pressure p_{c} for a prescribed initial concentration c_0 ($p_{\mathrm{m}} > p_{\mathrm{c}} = p_{\mathrm{a}}c_0^2/k_{\mathrm{s}}^2$), magma represents a homogeneous viscous fluid with viscosity (1). Lying above is the nucleation zone (zone 4) in which bubbles are formed at $p_{\mathrm{m}} \sim p_{\mathrm{c}}$. It is assumed that bubbles are not formed and do not disappear beyond the nucleation zone. According to the evaluations of [15], the length of the nucleation zone is substantially smaller than the length of the channel; therefore, we can consider it as the surface. In the central region (region 3) where $p_{\mathrm{m}} < p_{\mathrm{c}}$, there flows a bubble liquid; here, by virtue of the higher viscosity bubbles are "frozen" into the magma and the pressure in the bubbles differs from the magma pressure due to viscous stresses. As the magma rises and the pressure in it drops, bubbles grow because of degassing. We have flow of a gas suspension resulting from the destruction (fragmentation) of the bubble medium in the upper portion of the channel (region 1). The fragmentation zone (zone 2) will be considered as a surface analogously to the nucleation zone.

Let us describe the motion of magma in the region of a homogeneous regime. As is shown in [12], for characteristic values of c_0 and k_{s} we can disregard a change in the velocity of rise of the magma. If we additionally disregard the resistance in the volcanic channel, in the region of homogeneous flow we have

$$p_{\mathrm{m}} = p_0 - \rho_0 gz\,, \quad v_{\mathrm{m}} = v_0\,, \quad c_{\mathrm{m}} = c_0\,. \tag{3}$$

When the pressure of the homogeneous magma drops to the value of the saturation pressure p_{c}, we have transition to a nucleate (bubble) regime. We will assume that n_0 bubbles of radius a_0 instantaneously arise in a unit volume of the magma. The system of equations of the nucleate regime in dimensionless variables has the form [13, 16]

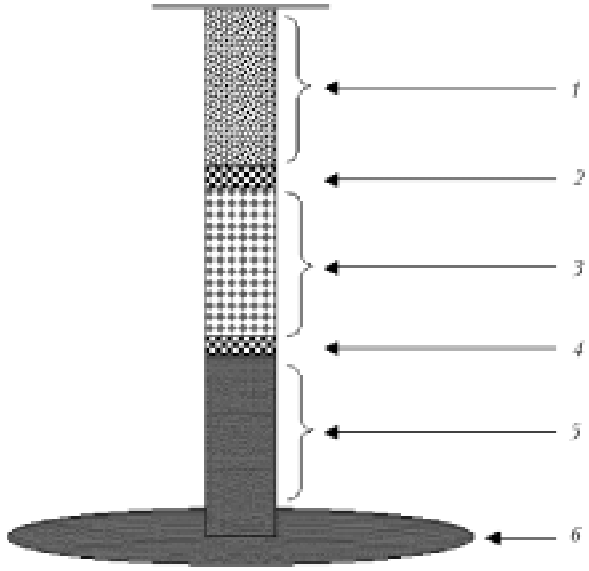

Fig. 1. Diagram of the distribution of difference regimes of flow of eruption products in the volcanic channel: 1) gas suspension; 2) fragmentation zone; 3) nucleate regime; 4) nucleation zone; 5) region of a homogeneous magma; 6) magmatic source.

$$(1-\alpha'_b)\,(1-c_m)\,v'_b = 1-c_0\,, \quad (\rho'_g\alpha'_b\delta + (1-\alpha'_b)\,c_m)\,v'_b = c_0\,, \quad n'_b v'_b = 1\,,$$

$$v'_b\,\frac{d\rho'_g}{dz'} = \frac{3}{2\,\mathrm{Pe}}\left[\frac{c_m - c_0\sqrt{p'_g}}{a'_b\delta} - \mathrm{Pe}\,\rho'_g v'_b\,\frac{da'_b}{dz'}\right], \quad v'_b\,\frac{da'_b}{dz'} = \frac{C_a}{4\mu'}\,a'_b\,(p'_g - p'_m)\,,$$

$$\frac{dp_t}{dz'} = -\rho_t - F_w\,, \quad p_t = (1-\alpha'_b)\,p'_m + \alpha'_b p'_g\,, \quad \rho_t = \alpha'_b\delta\rho'_g + (1-\alpha'_b)\,, \tag{4}$$

$$F_w = \varepsilon\,\mathrm{Ar}\,\mu' v'_b\,, \quad \alpha'_b = (a'_b)^3\,n'_b\,, \quad p'_g = \rho'_g\,, \quad C_a = \frac{p_0^2}{\rho_0 g \mu_0 v_0}\,.$$

The dimensional quantities are expressed by dimensionless ones as follows:

$$z = z'\,\frac{p_0}{\rho_0 g}\,, \quad p_m = p'_m p_0\,, \quad p_g = p'_g p_0\,, \quad \rho_m = \rho'_m \rho_0\,,$$

$$\rho_g = \rho'_g \rho_g^0\,, \quad n_b = n'_b n_0\,, \quad a_b = a'_b a_0\,, \quad v_b = v'_b v_0\,, \quad \mu = \mu'\mu_0\,, \quad \alpha_b = \alpha'_b \alpha_b^0\,. \tag{5}$$

Here

$$\alpha_b^0 = a_0^3 n_0\,, \quad \rho_g^0 = \frac{p_0\eta}{RT_0}\,, \quad \delta = \frac{\rho_g^0}{\rho_0} = \frac{p_0\eta}{RT_0\rho_0}\,, \quad \mathrm{Ar} = \frac{\mu_0 v_0}{\rho_0 g d^2}\,, \quad \mathrm{Pe} = \frac{\rho_0 g v_0}{D p_0 a_0^2}\,. \tag{6}$$

The temperature of the bubble liquid T_0 is set constant.

We will assume that the transition from the bubble liquid to a gas suspension occurs when the volume concentration of bubbles attains the critical value α_c. We write the laws of conservation of mass, energy, and momentum for the fragmentation zone:

$$\rho_2\alpha_2 v_2 = v_b (1 - \alpha_b) \rho_m , \quad \rho_1 v_1 (1 - \alpha_2) = v_b \alpha_b \rho_g ,$$

$$p_1 + \rho_1 (1 - \alpha_2) (v_1)^2 + \rho_2 \alpha_2 (v_2)^2 = (\rho_m (1 - \alpha_b) + \rho_g \alpha_b) (v_b)^2 + p_m (1 - \alpha_b) + p_g \alpha_b ,$$

$$p_1 ((1 - \alpha_2) v_1 + \alpha_2 v_2) + \rho_1 v_1 (1 - \alpha_2) \left(s_g T_1 + \frac{(v_1)^2}{2} \right) + \tag{7}$$

$$+ \rho_2 \alpha_2 v_2 \left(s_m T_2 + \frac{(v_2)^2}{2} \right) = (\rho_m (1 - \alpha_b) + \rho_g \alpha_b) v_b \frac{(v_b)^2}{2} +$$

$$+ (\rho_m (1 - \alpha_b) s_m + \rho_g \alpha_b s_g) T_0 v_b + (p_m (1 - \alpha_b) + p_g \alpha_b) v_b .$$

The last two equations of system (7) represent energy- and momentum-conservation equations for the entire mixture rather than for the condensed and gaseous components separately, since to compute them we must know the values of the exchange terms in the process of destruction, which substantially depend on the mechanism of the process and are unknown. Instead, let us suppose, following [15], that a fluidization condition is attained at exit from the fragmentation zone, i.e., the gas velocity relative to particles is such that the resistance force acting on the particles is equal to their weight. Furthermore, let us suppose that the density and temperature of the magma before the fragmentation zone are equal to the density and temperature of the condensed gas-suspension particles and the gas constant, while the gas suspension obeys the Mendeleev–Clapeyron equation.

Thus, system (7) is supplemented with the equations

$$\frac{9}{2} \frac{\mu_v}{a_d^2} (v_1 - v_2) = \rho_2 g , \quad \rho_2 = \rho_m , \quad T_2 = T_0 , \quad p_1 = \frac{\rho_1}{\eta} R T_1 \tag{8}$$

and becomes closed.

We used the model employed in [21] to describe the motion of a gas-dust suspension in the upper part of a volcanic channel and in the atmosphere.

It is assumed that the suspension is monodisperse and the dust particles are spherically shaped and possess heat capacity characteristic of solid products of volcanic eruptions. The volcanic-channel cross section is considered to be cylindrical.

In the case of axial symmetry of the eruption, the gasdynamic flow of eruption products is described by the system of equations of gas-suspension mechanics [23]

$$\frac{\partial (\alpha_1 \rho_1)}{\partial t} + \frac{\partial}{\partial z} (v_1 \alpha_1 \rho_1) + \frac{1}{r} \frac{\partial}{\partial r} (r u_1 \alpha_1 \rho_1) = 0 , \quad \frac{\partial (\alpha_2 \rho_2)}{\partial t} + \frac{\partial}{\partial z} (v_2 \alpha_2 \rho_2) + \frac{1}{r} \frac{\partial}{\partial r} (r u_2 \alpha_2 \rho_2) = 0 ,$$

$$\frac{\partial (\alpha_1 \rho_1 v_1)}{\partial t} + \frac{\partial (\alpha_1 \rho_1 v_1^2)}{\partial z} + \frac{1}{r} \frac{\partial (r \alpha_1 \rho_1 v_1 u_1)}{\partial r} = - \left(1 - \frac{3}{2} \alpha_2 \right) \frac{\partial p_1}{\partial z} - \left(1 - \frac{3}{2} \alpha_2 \right) n_d f_z + \left(1 - \frac{\alpha_2}{2} \right) \alpha_1 \rho_1 g ,$$

$$\frac{\partial (\alpha_1 \rho_1 u_1)}{\partial t} + \frac{\partial (\alpha_1 \rho_1 u_1 v_1)}{\partial z} + \frac{1}{r} \frac{\partial (r \alpha_1 \rho_1 u_1^2)}{\partial r} = - \left(1 - \frac{3}{2} \alpha_2 \right) \frac{1}{r} \frac{\partial (r p_1)}{\partial r} - \left(1 - \frac{3}{2} \alpha_2 \right) n_d f_r ,$$

$$\frac{\partial (\alpha_2 \rho_2 v_2)}{\partial t} + \frac{\partial (\alpha_2 \rho_2 v_2^2)}{\partial z} + \frac{1}{r} \frac{\partial (r \alpha_2 \rho_2 v_2 u_2)}{\partial r} = - \frac{3}{2} \alpha_2 \frac{\partial p_1}{\partial z} + \left(1 - \frac{3}{2} \alpha_2 \right) n_d f_z + \alpha_2 \rho_2 g + \frac{1}{2} \alpha_2 \alpha_1 \rho_1 g ,$$

$$\frac{\partial (\alpha_2 \rho_2 u_2)}{\partial t} + \frac{\partial (\alpha_2 \rho_2 u_2 v_2)}{\partial z} + \frac{1}{r} \frac{\partial (r \alpha_2 \rho_2 u_2^2)}{\partial r} = - \frac{3}{2} \alpha_2 \frac{1}{r} \frac{\partial (r p_1)}{\partial r} + \left(1 - \frac{3}{2} \alpha_2 \right) n_d f_r ,$$

(9)

$$\frac{\partial(\alpha_1\rho_1 e_1)}{\partial t}+\frac{\partial(\alpha_1\rho_1 v_1 e_1)}{\partial z}+\frac{1}{r}\frac{\partial(r\alpha_1\rho_1 e_1 u_1)}{\partial r}-\frac{\alpha_1 p_1}{\rho_1}\left[\frac{\partial\rho_1}{\partial t}+\frac{\partial(\rho_1 v_1)}{\partial z}+\frac{1}{r}\frac{\partial(r\rho_1 u_1)}{\partial r}\right]=$$

$$=\left[\left(1-\frac{3}{2}\alpha_2\right)n_d f_z+\frac{1}{2}\alpha_2\left(\alpha_1\rho_1 g-\frac{\partial p_1}{\partial z}\right)\right]w_z+\left[\left(1-\frac{3}{2}\alpha_2\right)n_d f_r-\frac{1}{2}\alpha_2\frac{1}{r}\frac{\partial(rp_1)}{\partial r}\right]w_r+n_d q_1\,,$$

$$\frac{\partial(\alpha_2\rho_2 e_2)}{\partial t}+\frac{\partial(\alpha_2\rho_2 v_2 e_2)}{\partial z}+\frac{1}{r}\frac{\partial(r\alpha_2\rho_2 e_2 u_2)}{\partial r}=n_d q_2\,,$$

$$q_1=2\pi a_d\,\mathrm{Nu}_1\lambda_1(T_2-T_1)\,,\quad q_2=2\pi a_d\,\mathrm{Nu}_2\lambda_2(T_1-T_2)\,,\quad \alpha_1+\alpha_2=1\,,$$

$$\frac{n_d f_z}{\alpha_2}=\frac{9}{2}\frac{\mu_v}{a_d^2}w_z\,,\quad \frac{nf_r}{\alpha_1}=\frac{9}{2}\frac{\mu_v}{a_d^2}w_r\,.$$

System (9) is supplemented with equations of state in the form

$$e_1=\frac{p_1}{(m-1)\rho_1}\,,\quad e_2=\int_{T_0}^{T}s_m(T)\,dT\,,\quad p_1=\frac{R}{\eta}\rho_1 T_1\,. \tag{10}$$

The quantities $(\alpha_1\rho_1)$ and $(\alpha_2\rho_2)$ employed in (9) represent the reduced densities of the gas and the dust (masses of the corresponding phases in a unit volume of the mixture).

Remark. In calculating the steady-state flows of the gas-dust suspension in the upper part of the volcanic channel, one must take into account the fact that if the flow is subsonic at exit from the channel, the exit pressure is equal to atmospheric pressure; otherwise, as a boundary condition, it is necessary to equalize the flow velocity and the velocity of sound.

Procedure of Calculation and Processing of the Results. Let us describe the method of calculation of steady-state flow of eruption products in the atmosphere and of the distribution of different regions of motion of the magma and the gas suspension in the volcanic channel. For computations we must know the values of the following parameters: 1) pressure in the source p_0, 2) initial concentration of the gas in the melt c_0, 3) parameter in the empirical law of solubility $k_s/\sqrt{p_a}$, 4) initial velocity of rise of the magma v_0, 5) viscosity amplitude μ_0, 6) magma density ρ_0, 7) length of the volcanic channel H, 8) radius of the volcanic channel r_0, 9) critical pressure of transition to a nucleate regime p_c, and 10) critical value of the volume concentration of bubbles α_c.

Setting the values of the parameters of the magma and the source, we easily determine what kind of flow regime we will have in the lower part of the neck: if the pressure of the magma flowing out of the source is higher than the saturation pressure of volatile components dissolved in the magma p_c, a homogeneous magma flows out of the source and it becomes a bubble liquid as soon as the pressure drops to the value of p_c. The coordinate of transition is determined from Eq. (3):

$$z_c=\frac{p_0-p_c}{\rho_1 g}\,. \tag{11}$$

If the depth of the volcanic channel H is such that the pressure fails to attain the value of p_c before the channel emerges, then a homogeneous magma flows out of the volcanic neck.

The parameters of steady-state flow of a bubble liquid were calculated by the Runge–Kutta method of fourth order.

According to the method, we assumed in the calculations that the transition layer between the bubble liquid and the gas suspension is at a depth where the volume concentration of bubbles α attains the critical value α_c. Calculation according to the algorithm described above enables us to find the depth of location of the fragmentation zone, setting the value of α_c. If the value of α does not attain α_c over the entire length of the channel, this indicates that

TABLE 1. Values of the Parameters of a Magmatic Source and the Characteristics of Magma Employed in the Examples of Calculations

Quantity	Calculation I	Calculation II	Quantity	Calculation I	Calculation II
p_0	$2.5 \cdot 10^8$	$2.56 \cdot 10^8$	ρ_0	5000	5000
c_0	0.08	0.08	H	11 000	7500
$k_s/\sqrt{p_a}$	$1 \cdot 10^{-5}$	$0.5 \cdot 10^{-5}$	r_0	300	300
v_0	15	15	p_c	$1 \cdot 10^8$	$6.4 \cdot 10^7$
μ_0	$1 \cdot 10^4$	$1 \cdot 10^4$	α_c	0.83	0.87

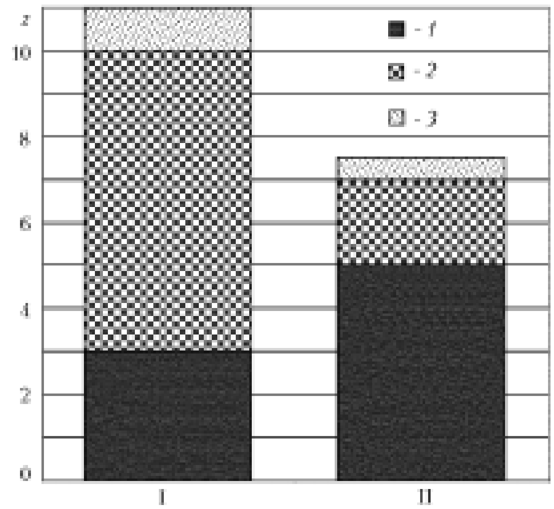

Fig. 2. Lengths of the regions of flow of a homogeneous magma (1), a bubble liquid (2), and a gas suspension (3) in the volcanic channel for the examples considered (I and II). z, km.

a bubble liquid flows out of the volcanic neck in the steady-state regime of eruption for the given set of parameters. The surface parameters of the liquid are determined according to the same algorithm.

Gas-suspension flow in the volcanic channel and in the atmosphere was calculated by the target method for the prescribed position of the beginning of the gas-suspension zone relative to the magmatic source.

At each calculation step, we prescribed, by the target method, the assumed length of the gas-suspension region H_d; next we solved system (9)–(10) with fixed boundary conditions on the fragmentation wave (the conditions had been obtained by application of system (7)–(8) to the bubble-liquid parameters computed earlier). We specified non-flow conditions on the channel walls and the earth's surface.

We performed computations until the process became steady-state (usually for about 300 sec). Thereafter the value of H_d changed and the next step began. The criterion of completion of calculation by the target method was finding a value of H_d such that the velocity of the gas component of the mixture at exit to the atmosphere was equal to the velocity of sound (see [16] and the remark at the end of the previous section).

System (7)–(8) was solved by the large-particle method [24] according to the scheme employed in [21].

Thus, by prescribing the flow rate and properties of the magma and the diameter of the channel, we can calculate, by the scheme described, the neck length corresponding to these parameters, the steady-state distribution of the regions of flow of the homogeneous magma, the bubble liquid, and the gas-suspension, and also the values of the characteristics of the eruption products throughout the volcanic channel.

Given below are two examples of numerical calculations according to the algorithm described.

If we know the depth of the volcanic neck and the properties of the magmatic melt, while the magma flow rate is unknown, we must also use the target method to calculate the steady-state regime of eruption. The above-described problem of finding the length of the neck and the parameters of eruption products is solved at each step on the basis of a certain assumed value of the magma flow rate. The obtained value of the channel length is compared

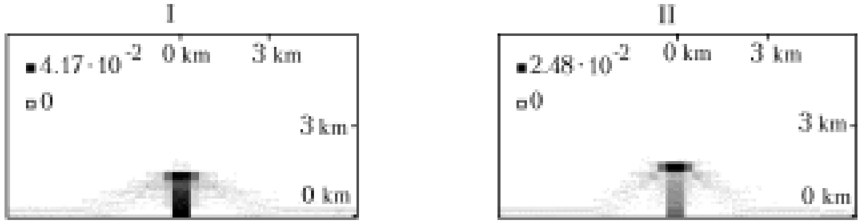

Fig. 3. Quasistationary distribution of the volume concentration of a volcanic dust in the atmosphere α_2 for the examples considered (I and II). The gray-color intensity is in proportion to α_2.

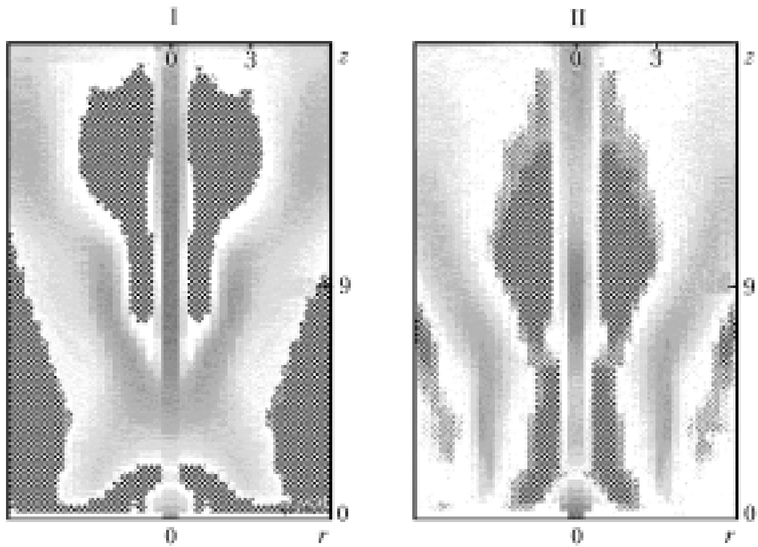

Fig. 4. Quasistationary distribution of the vertical component of the gas velocity u_1 in the atmosphere for the examples considered (I and II). The gray-color intensity is in proportion to u_1, when it is positive (upward direction). In the hatched regions, the quantity u_1 is negative (regions of descending motion of the gas).

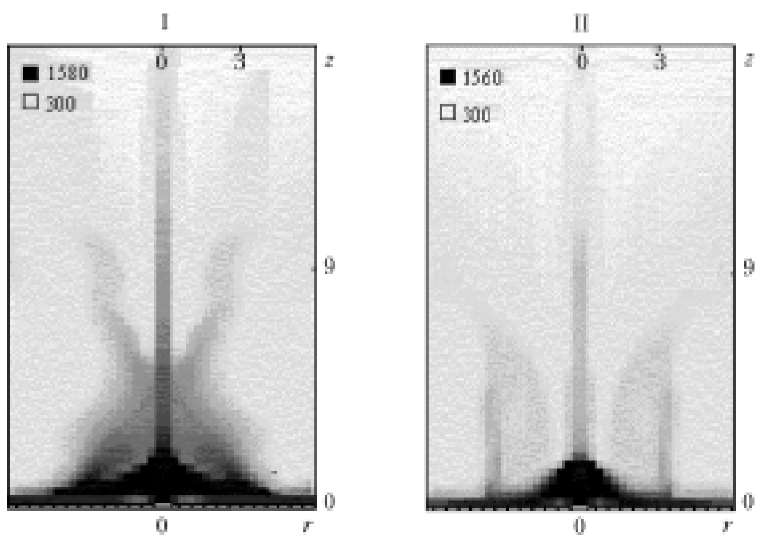

Fig. 5. Quasistationary distribution of the gas temperature T_1 in the atmosphere for the examples considered (I and II). The gray-color intensity is in proportion to T_1. z and r, km.

to that prescribed and accordingly the value of the magma flow rate is changed, after which the next iteration is performed. As a result, we will find the steady-state flow rate and parameters of flow of the eruption products for the given depth of the neck and properties of the magma.

Examples of Calculation. We give two examples of numerical calculations of the steady-state flow of eruption products in the volcanic channel and the atmosphere based on the model described. The values of the parameters of the magmatic source and the characteristics of magma employed in calculating are given in Table 1. In the computations, we employed the requirement of equality of the velocity of the gas on emergence to the velocity of sound.

Subdivision of the volcanic channel into the regions of homogeneous flow, a bubble liquid, and a gas suspension for the cases under study is given in Fig. 2.

The parameters of the atmosphere and the gas-dust suspension above the volcano were determined in the region $0 < r < R_r$ and $0 < z < H_r$. At the boundaries $z = H_r$ and $r = R_r$, the parameters of the gas were considered to be equal to the parameters of an undisturbed atmosphere.

Results of the calculations of the flow-parameter distribution in the atmosphere are shown in Figs. 3–5. Figure 3 gives the steady-state distribution of the volume concentration of dust in the atmosphere. It follows from the figure that the height of the dust column in steady-state eruption is more than a kilometer.

An analogous map for the vertical component of the gas velocity is shown in Fig. 4, where the presence of the vortices in the steady-state flow is obvious.

Figure 5 shows the temperature distribution of the gas phase of the gas suspension.

CONCLUSIONS

The constructed model can be used for evaluation of the parameters of the atmosphere above an erupting volcano, the height of a dust column during the quasistationary stage of eruption, the thickness of the layer of ash fallen on the earth, and other forms of action of volcanic eruptions on the environment. The model can easily be corrected in order to allow for the influence of atmospheric phenomena (wind, cyclones, etc.) and the surface features of the adjacent territories on the process of eruption.

NOTATION

a_0, radius of bubbles arising when magma goes to the state of a bubble liquid, m; a_b, radius of gas bubbles in the magma, m; a'_b, dimensionless radius of gas bubbles in the magma; a_d, radius of dust particles, m; A and B, constants dependent on the magma properties (A, Pa·m^3/mole; B, dimensionless); Ar, Archimedes number; c_0, volume concentration of the gas dissolved in the magma at exit from the source, dimensionless constant; c_m, running volume concentration of the gas dissolved in the magma in the channel at a given depth, dimensionless variable; C_a, dimensionless constant; d, diameter of the volcanic channel, m; D, coefficient of diffusion, m^2/sec; e_1 and e_2, internal energy of a unit mass of the gas and the dust, J/kg; $\mathbf{f} = (f_r, f_z)$, bulk force of interphase interaction, N/m^3; f_r and f_z, projections of the force onto the r and z directions respectively; F_w, dimensionless force of resistance of channel walls; g, free-fall acceleration, m/sec^2; H, length of the volcanic channel, m; H_r, height of the computational region, m; H_d, length of the gas-suspension region, m; m, dimensionless adiabatic exponent of the gas; n_0, number of gas bubbles in a unit volume at the instant of formation of a bubble liquid, m^{-3}; n_b, number of gas bubbles in a unit volume, m^{-3}; n'_b, dimensionless number of gas bubbles in a unit volume; n_d, volume concentration of the dust, m^{-3}; Nu_1 and Nu_2, dimensionless Nusselt numbers of the gas and the dust; p_0, pressure of the magma in the magnetic source, Pa; p_1, pressure of the gas in the gas suspension, Pa; p_a, atmospheric pressure on the earth's surface, Pa; p_c, saturation pressure — pressure of the homogeneous-to-nucleate transition of flow, Pa; p_g, pressure of the gas in bubbles, Pa; p'_g, dimensionless pressure of the gas in bubbles; p_m, magma pressure, Pa; p'_m, dimensionless pressure of the magma in the bubble liquid; k_s, dimensionless constant, parameter in the solubility law; p_t, dimensionless "total" (weighted) pressure of the bubble liquid; Pe, dimensionless Péclet number; r, radial space variable of the cylindrical coordinate system, m; r_0, radius of the volcanic channel, m; R, gas constant, J/(kmole·K); R_r, radius of the computational region, m; q_1 and q_2, heat fluxes through the particle surface to the gas medium and into the particles respectively, J/sec; s_m, specific heat of the magma substance, J/sec; s_g, specific heat of the gas, J/(kg·K); t, time, sec; T, temperature of the magma in the channel, K; T_0, temperature of the magma in the magmatic source, K; T_1 and T_2, temperatures of the gas and the dust respectively in the gas suspension, K; u_1 and u_2, horizontal components of the velocity of the gas and the dust (in the gas suspension), m/sec; v_0, velocity of rise of the magma flowing out of the source, m/sec; v_1 and v_2, vertical components of the velocity of the gas and the dust (in the gas suspension), m/sec; v_b, velocity of rise of the bubble liquid, m/sec; v'_b, dimensionless velocity of rise of the bubble liquid; v_m, velocity of rise of the homogeneous magma, m/sec; $\mathbf{w} = (w_r, w_z) = (u_1 - u_2, v_1 - v_2)$, velocity of the gas relative to the particles of dust, m/sec; w_r and w_z, projections onto the r and z directions respectively; z, vertical coordinate; z', dimensionless vertical coordinate; z_c, coordinate of transition to a nucleate regime, m; α_1 and α_2, dimensionless volume fractions of the gas and the dust in the gas suspension; α_c, dimensionless critical value of the concentration of gas bubbles in the bubble liquid at which transition to a gas-suspension phase occurs; α_b^0, dimensionless constant for normalization of the volume fraction of the gas in the bubble liquid; α_b, dimensionless fraction of the gas in the bubble liquid; α'_b, dimensionless normalized volume fraction of the gas in the bubble liquid; δ, dimensionless parameter; ε, dimensionless coefficient; η, molar mass of the gas phase, kg/kmole; λ_1 and λ_2, thermal conductivities of the gas and the dust, J/(m·sec·K); μ, coefficient of viscosity of magma, Pa·sec; μ', dimensionless coefficient of viscosity of magma; μ_0, coefficient of viscosity of the "dry" magma, Pa·sec; μ_v, coefficient of dynamic viscosity of the gas in the atmosphere, kg/(m·sec); ρ_0, density of the magma in the magmatic source, kg/m^3; ρ'_g, dimensionless density of the gas in bubbles; ρ_t, dimensionless "total" (weighted) density of the bubble liquid; ρ_g^0, constant for normalization of the density of the gas in bubbles in the bubble liquid, kg/m^3; ρ_g, density of the gas in bubbles, kg/m^3; ρ_m, density of magma, kg/m^3; ρ_1 and ρ_2, densities of the gas and the dust respectively in the gas suspension, kg/m^3. Subscripts: a, atmospheric; b, bubble; c, critical; d, dust; g, gas; m, magma; r, region; s, solubility; t, total; v, viscosity; w, wall.

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